TAMKANG JOURNAL OF MATHEMATICS
Volume 48, Number 2, 149-157, June 2017
doi:10.5556/j.tkjm.48.2017.2295
This paper is available online at http://journals.math.tku.edu.tw/index.php/TKJM/pages/view/onlinefirst

ON GLOBAL DOMINATING-χ-COLORING OF GRAPHS

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Abstract. Let G be a graph. Among all χ-colorings of G, a coloring with the maximum number of color classes that are global dominating sets in G is called a global dominating-χ-coloring of G. The number of color classes that are global dominating sets in a global dominating-χ-coloring of G is defined to be the global dominating -χ- color number of G, denoted by $gd_\chi(G)$. This concept was introduced in [5]. This paper extends the study of this notion.

1. Introduction

By a graph $G = (V, E)$, we mean a connected, finite, non-trivial, undirected graph with neither loops nor multiple edges. The order and size of G are denoted by n and m respectively. For graph theoretic terminology we refer to Chartand and Lesniak [3].

A subset D of vertices is said to be a *dominating set* of G if every vertex in V either belongs to D or is adjacent to a vertex in D. The *domination number* $\gamma(G)$ is the minimum cardinality of a dominating set of G . A subset D of vertices is said to be a *global dominating set* of G if D is a dominating set of both G and $\overline{G}$; that is, every vertex outside D has a neighbour as well as a non-neighbour in D. The *global domination number* $\gamma_g(G)$ is the minimum cardinality of a global dominating set of G.

A proper coloring of a graph G is an assignment of colors to the vertices of G in such a way that no two adjacent vertices receive the same color. Since all colorings in this paper are proper colorings, we simply call a proper coloring a coloring. A coloring in which k colors are used is a *k-coloring*. The *chromatic number* of G, denoted by $\chi(G)$, is the minimum integer k for which G admits a *k-coloring*. In a given coloring of the vertices of a graph G, a set consisting of all those vertices assigned the same color is called a *color class*. If $\mathscr{C}$ is a coloring of G with the color classes $U_1, U_2, \ldots, U_t$, then we write $\mathscr{C} = \{U_1, U_2, \ldots, U_t\}$. Among all *$\chi$-colorings* of G, let $\mathscr{C}$ be chosen to have a color class U that dominates as many vertices of G as possible. If there is a vertex in G not dominated by U, then deleting such a vertex from

Received September 4, 2016, accepted February 24, 2017.
2010 *Mathematics Subject Classification*. 05C15, 05C69.
Key words and phrases. Dominating-χ-coloring, global dominating-χ-coloring.
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its color class and adding it to the color class U produces a new minimum vertex-coloring that contains a color class which dominates more vertices than U, a contradiction. Hence the color class U dominates G. Thus we have the following observation first observed in [1].

Observation 1.1. *Every graph G contains a χ-$coloring$ with the property that at least one color class is a dominating set in G.*

Motivated by Observation 1.1, Arumugam et al. [1] defined the dominating -χ- color number, which they called dom-color number, as follows. Among all χ-colorings of G, a coloring with the maximum number of color classes that are dominating sets in G is called a *dominating-χ-coloring* of G. The number of color classes that are dominating sets in a dominating-χ-coloring of G is defined to be the *dominating -χ- color number* of G, denoted by $d_\chi(G)$. This parameter has been further studied in [2] and [4].

In [5], the notion of dominating-χ-coloring was extended to the notion of global dominating sets in the name of global dominating-χ-coloring. Among all χ-colorings of G, a coloring with the maximum number of color classes that are global dominating sets in G is called a *global dominating-χ-coloring* of G. The number of color classes that are global dominating sets in a global dominating-χ-coloring of G is defined to be the *global dominating -χ- color number* of G and is denoted by $gd_\chi(G)$. Certainly, for any graph G, we have $d_\chi(G) \geq gd_\chi(G)$. In this paper, we discuss the parameter gd_χ for unicyclic graph and also prove some realization theorems associated with some relations among gd_χ, d_χ and χ.

We need the following theorems.

Theorem 1.2 ([2])**.** *For any graph G, we have $d_\chi(G) \leq \delta(G)+1$.*

Theorem 1.3 ([5])**.** *For any graph G, we have $gd_\chi(G) \leq \delta(G)+1$.*

Theorem 1.4 ([5])**.** *If G is a graph of order $n \geq 2$, then $gd_\chi(G) \leq \frac{n-\chi(G)s(G)}{\gamma_g(G)-s(G)}$, where $s(G)$ denotes the minimum cardinality of any color class in any χ-coloring of G.*

Theorem 1.5 ([5])**.** *If G is a graph with $\Delta(G) = n-1$, then $gd_\chi(G) = 0$.*

2. gd_χ for unicyclic graphs

Throughout the paper, by a unicyclic graph, we mean a connected unicyclic graph that is not a cycle. Now, in view of Theorem 1.3, for a graph with minimum degree 1, the value of global dominating χ - color number is at most 2. In particular, for a unicyclic graph G, $gd_\chi(G) \leq 2$. So, the family of unicyclic graphs can be classified into three classes namely graphs with $gd_\chi = 0$; graphs with $gd_\chi = 1$ and graphs with $gd_\chi = 2$. This section determines these classes of graphs. For this purpose, we describe the following families.

(i) Let $\mathscr{G}_1$ be the class of all connected unicyclic graphs obtained from a cycle of length 4 by attaching at least one pendant edge at exactly two adjacent vertices of the cycle. A graph in this family is given in Figure 1(a).

(ii) Let $\mathscr{G}_2$ be the collection of all connected unicyclic graphs obtained from a cycle of length 4 by attaching at least one pendant edge at each of two non adjacent vertices of the cycle. A graph lying in this family is given in Figure 1(b).

(iii) Let $\mathscr{G}_3$ be the collection of all connected unicyclic graphs obtained from a cycle of length 4 by attaching at least one pendant edge at each of any three vertices of the cycle. A graph lying in this family is given in Figure 1(c).

(iv) Let $\mathscr{G}_4$ be the collection of all connected unicyclic graphs with the cycle $C = (v_1, v_2, v_3, v_4, v_1)$ that are constructed as follows. Attach $r \geq 0$ pendant edges at v_1, $s \geq 0$ pendant edges at v_3. Also, attach $t \geq 1$ pendant edges at v_2, say $x_1, x_2, \ldots, x_t$ are the corresponding pendant vertices adjacent to v_2. Finally, for each $i \in \{1, 2, \ldots, t\}$, attach t_i pendant vertex at the vertex x_i with the condition that $t_1 \geq 1$ and $t_j \geq 0$ for all $j \neq 1$. A graph lying in this family is given in Figure 1(d).

(v) Let $\mathscr{G}_5$ be the family of connected unicyclic graphs obtained from a triangle by attaching at least one pendant edge at exactly one vertex of the triangle.

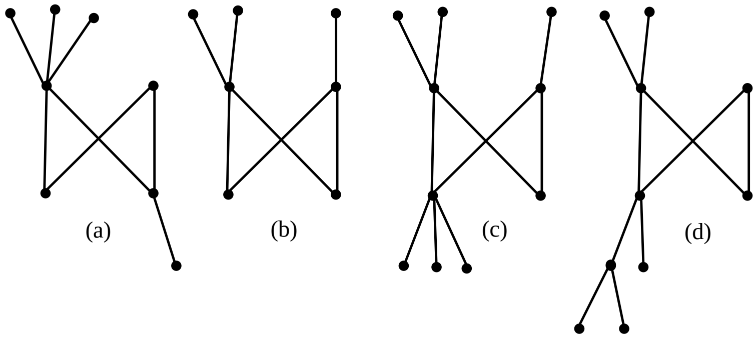

Figure 1: (a) A graph in $\mathscr{G}_1$, (b) A graph in $\mathscr{G}_2$, (c) A graph in $\mathscr{G}_3$, (d) A graph in $\mathscr{G}_4$.

Theorem 2.1. *Let G be a unicyclic graph with even cycle C. If C is of length at least 6, then $gd_\chi(G) = 2$.*

Proof. Certainly $\chi(G) = 2$. Let $\{V_1, V_2\}$ be the χ-coloring of G. Obviously, both V_1 and V_2 are dominating sets of G. It is enough to verify that V_1 and V_2 are global dominating sets of G. Since the length of the cycle C is at least 6, it follows that each of V_1 and V_2 contains at least three vertices of G lying on C. However, every vertex of G has at most two neighbours on C ; this means that every vertex of V_1 has a non-neighbour in V_2 and every vertex of V_2 has a non-neighbour in V_1. Thus V_1 and V_2 are global dominating sets of G. □

Theorem 2.2. *Let G be a unicyclic graph whose cycle is of length 4. Then $gd_\chi(G) = 0$ if and only if $G \in \mathcal{G}_1$.*

Proof. Let $C = (v_1, v_2, v_3, v_4, v_1)$ and let $\{X, Y\}$ be the χ-coloring of G. Assume that $v_1, v_3 \in X$ and $v_2, v_4 \in Y$. Obviously, both X and Y are dominating sets of G. Now, suppose $gd_\chi(G) = 0$. Then both X and Y can not be global dominating sets. Therefore there exist vertices $x \in X$ and $y \in Y$ such that x is adjacent to all the vertices of Y and y is adjacent to all the vertices of X. Since G is unicyclic, each of x and y must lie on C, say $x = v_1$ and $y = v_2$. Again, as G is unicyclic, the vertex v_4 is not adjacent to any vertex of X other than v_1 and v_3. Similarly, the vertex v_3 is not adjacent to any vertex of Y other than v_2 and v_4. Further, a vertex of $X - \{v_1, v_3\}$ can not be adjacent with any vertex of $Y - \{v_2, v_4\}$ and similarly a vertex of $Y - \{v_2, v_4\}$ can not be adjacent with any vertex of $X - \{v_1, v_3\}$; for otherwise a cycle distinct from C will get formed. That is, v_2 is the only neighbour in Y for each vertex of $X - \{v_1, v_3\}$ and v_1 is the only neighbour in X for each vertex of $Y - \{v_2, v_4\}$. Thus the vertices of G outside C are pendant and therefore $G \in \mathcal{G}_1$. The converse is an easy verification. □

Theorem 2.3. *Let G be a unicyclic graph whose cycle is of length 4. Then $gd_\chi(G) = 1$ if and only if $G \in \cup_{i=2}^{4} \mathcal{G}_i$.*

Proof. Let $\{V_1, V_2\}$ be the χ-coloring of G. Assume that V_2 is a global dominating set of G and V_1 is not. Also, assume that $v_1, v_3 \in V_1$ and $v_2, v_4 \in V_2$. As V_1 is not a global dominating set, there is a vertex $x \in V_2$ that is adjacent to all the vertices of V_1. As discussed in the proof of Theorem 2.2, x must lie on C. But V_2 is a global dominating set. Therefore, every vertex of V_1 has a non-neighbour in V_2 and so the set $B = V_2 - \{v_2, v_4\} \neq \phi$. Since v_2 is adjacent to every vertex of V_1, every vertex in B is a pendant vertex of G. Now, let A be the set of neighbours of v_2 in V_1 other than v_1 and v_3. If $A = \phi$, then $N(v_1) \cap B \neq \phi$ and $N(v_3) \cap B \neq \phi$ and $[N(v_1) \cup N(v_3)] \cap B = B$. Thus $G \in \mathcal{G}_1$.

Suppose $A \neq \phi$. Now, if the vertices in A are pendant, then $N(v_1) \cap B \neq \phi$, $N(v_3) \cap B \neq \phi$ and $[N(v_1) \cup N(v_3)] \cap B = B$ so that $G \in \mathcal{G}_2$. So, the remaining case is that $A \neq \phi$ and A has a vertex u with $deg\ u \geq 2$. That is, u has a neighbour in B, say w. Note that the vertex w is a non-neighbour of both v_1 and v_3 as w is pendant. But however the vertices v_1 and v_3 may have neighbours in B and thus $G \in \mathcal{G}_3$. Now, it is not difficult to see that if $G \in \cup_{i=2}^{4} \mathcal{G}_i$, then $gd_\chi(G) = 1$. □

Lemma 2.4. *If $gd_\chi(G) = 0$, then $d_\chi(G) \geq 2$.*

Proof. Suppose $gd_\chi(G) = 0$ and $d_\chi(G) = 1$. Consider a χ-coloring $\{V_1, V_2, \ldots, V_\chi\}$ of G such that V_1 is a dominating set of G. As $gd_\chi(G) = 0$, V_1 can not be a global dominating set of G. Therefore, there exists a vertex v such that v is adjacent to every vertex of V_1. Assume without loss of generality that $v \in V_2$. Certainly, no $V_i (2 \le i \le \chi)$, is a dominating set and in particular V_2 is not a dominating set. So, there are vertices in $V - V_2$ that are not dominated by any vertex of V_2 ; let S be the set of those vertices. Clearly $S \subseteq V - V_2$. Also, as v is adjacent to each vertex of V_1, it follows that $S \subseteq V - V_1$ and thus $S \subseteq V - (V_1 \cup V_2)$. Now, if D is an independent dominating set of the subgraph $\langle S \rangle$ induced by S, then $V_2 \cup D$ is an independent dominating set of G. Therefore $\{V_1, V_2 \cup D, V_3 - V_3', V_4 - V_4', \ldots, V_\chi - V_\chi'\}$, where $V_i' = V_i \cap D$ for all $i \in \{3, 4, \ldots, \chi\}$ is a χ-coloring of G in which both V_1 and $V_2 \cup D$ are dominating sets of G, a contradiction to the assumption that $d_\chi(G) = 1$. □

Corollary 2.5. *If $d_\chi(G) = 1$, then $gd_\chi(G) = 1$.*

Let us now concentrate on the unicyclic graphs with odd cycle.

Theorem 2.6. *Let G be a unicyclic graph with odd cycle C. If all the vertices on C are support vertices, then $gd_\chi(G) = 1$.*

Proof. Let $C = (v_1, v_2, \ldots, v_n, v_1)$, where each v_i is support. In view of Corollary 2.5, it is enough to prove that $d_\chi(G) = 1$. As in Observation 1.1, $d_\chi(G) \ge 1$. For the other inequality, we need to prove that every χ-coloring of G has exactly one color class that is a dominating set of G. On the contrary, assume that G has a χ-coloring $\{V_1, V_2, V_3\}$ of G with V_1 and V_2 are dominating sets of G. It is clear that if x is a support vertex of G, then a dominating set of G must contain either x or all its pendant neighbours. Here V_1 and V_2 are assumed to be dominating sets and therefore all the support vertices and the pendant vertices of G must be contained in $V_1 \cup V_2$. In particular, $\{v_1, v_2, \ldots, v_n\}$ is a subset of $V_1 \cup V_2$; this is possible only when n is even. But n is odd and thus exactly one color class of any χ-coloring of G can be a dominating set of G. This completes the proof. □

Theorem 2.7. *Let G be a unicyclic graph with odd cycle C. If the length of C is at least 7 with the property that not all the vertices on C are supports, then $gd_\chi(G) = 2$.*

Proof. As we know $gd_\chi(G) \le 2$ and so in order to prove the theorem it is enough if we are able to come up with a χ-coloring of G where two color classes are global dominating sets. Here we provide such a coloring as follows. Let $C = (v_1, v_2, \ldots, v_n, v_1)$. Assume that v_1 is not a support vertex of G. Consider the χ-coloring $\{V_1, V_2\}$ of the tree $G - v_1 v_n$. Assume that $v_1 \in V_1$. Then $v_n \in V_1$. Now, take $\mathscr{C} = \{V_1 - \{v_1\}, V_2, \{v_1\}\}$. Then $\mathscr{C}$ is a χ-coloring of G. We prove that $V_1 - \{v_1\}$ and V_2 are global dominating sets of G. Note that both V_1 and V_2 are dominating sets of $G - v_1 v_n$. Therefore, obviously V_2 is a dominating set of G as well. Further, the set $V_2 - \{v_1\}$ also serves as a dominating set of G as v_1 is not a support. So, $V_1 - \{v_1\}$ and V_2 are dominating sets of G. Also, as the length of C is at least 7, it follows that each of $V_1 - \{v_1\}$ and V_2 contains at least three vertices of G lying on C. But every vertex of G can have at most two neighbours on C. So, every vertex of G will have a non-neighbour in each of $V_1-\{v_1\}$ and V_2 and therefore these two sets are global dominating sets of G. Thus $\mathscr{C}$ is a χ-coloring of G where $V_1-\{v_1\}$ and V_2 are global dominating sets of G as desired. □

Theorem 2.8. *Let G be a unicyclic graph whose cycle is of length* 5*. Then* $gd_\chi(G)$ *is either* 1 *or* 2.

Proof. Let $C=(v_1,v_2,v_3,v_4,v_5,v_1)$. Since G is a unicyclic graph, at least one of v_1, v_2, v_3, v_4 and v_5 has degree at least 3. Let it be v_1. Consider a neighbour u of v_1 outside C. Let $T=G-v_1v_5$. Then $\{V_1,V_2\}$ be a χ-coloring of G. Note that the vertices u, v_2 and v_4 belong to the same color class, say V_1. Then v_1, v_3 and v_5 belong to V_2. Certainly, $\{V_1, V_2-\{v_5\},\{v_5\}\}$ is a χ-coloring of G. We now claim that V_1 is a global dominating set of G. Clearly V_1 is a dominating set of G. Consider an arbitrary vertex x of G. If $x\in N[u]$, then v_4 is a non-neighbour of x. If $x\notin N[u]$, then u is a non-neighbour of x and so V_1 is a global dominating set of G. Hence $gd_\chi(G)\geq 1$. □

By an *extreme vertex* in a unicyclic graph G ; we mean a vertex v on the cycle C of G with the property that v is adjacent to a vertex outside C where degree is at least two. Let w be a vertex of G with $deg\ w\geq 3$. A *branch* of G at w is a maximal subtree T of G containing an edge outside C that is incident at w such that w is a pendant vertex in T.

Theorem 2.9. *Let G be a unicyclic graph whose cycle C is of length exactly* 3*. Then* $gd_\chi(G)=0$ *if and only if* $G\in\mathscr{G}_5$.

Proof. Let $C=(v_1,v_2,v_3,v_1)$. Assume $gd_\chi(G)=0$. We first prove that G has no extreme vertex. On the contrary, assume that G has an extreme vertex ; let it be v_1. Choose a vertex x in a branch of G at v_1 such that $d(v_2,x)=3$. Consider the χ-coloring $\mathscr{C}=\{V_1,V_2\}$ of the tree $G-v_1v_2$. As the distance between v_2 and x in G is 3, the distance between them in $G-v_1v_2$ is 4 and therefore they both belong to the same color class in $\mathscr{C}$, say V_1. Therefore $v_3\in V_2$ and $v_1\in V_1$. We now prove that there is a χ-coloring of G in which at least one color class is a global dominating set of G. If v_1 is not a support vertex, then consider the χ-coloring $\{V_1-\{v_1\},V_2,\{v_1\}\}$ of G. On the other hand, if v_1 is a support vertex, the consider the χ-coloring $\{(V_1-\{v_1\})\cup U, V_2-U,\{v_1\}\}$ of G, where U is the set of all pendant neighbours of v_1 (Note that U is a subset of V_2 in $\mathscr{C}$). Also remain that both x and v_2 belong to V_1. We now prove that $V_1-\{v_1\}$ and $(V_1-\{v_1\})\cup U$ are global dominating sets of G. Clearly both are dominating sets of G. Now, choose an arbitrary vertex y in G. If $y\in N[v_2]$, then x is a non-neighbour of y in V_1. If $y\notin N[v_2]$, then v_2 is a non-neighbour of y in V_1. This proves the result and so $gd_\chi(G)\geq 1$, a contradiction. Therefore G has no extreme vertex. That is, every vertex outside C is a pendant vertex and every vertex on C is either a support vertex or it is of degree exactly two.

Now, suppose exactly two vertices on C are support vertices, say v_2 and v_3. Then $\{S \cup \{v_1\}, \{v_2\}, \{v_3\}\}$, where S is the set of all pendant vertices of G, is a χ-coloring of G in which $S \cup \{v_1\}$ is a global dominating set of G and so $gd_\chi(G) \geq 1$, a contradiction. Suppose all the three vertices on C are support vertices. Then by Theorem 2.6, $gd_\chi(G) = 1$, again a contradiction. Hence the result. The converse follows from Theorem 1.5. □

3. Realization Theorems

Theorem 3.1. *For given integers k and l with $0 \leq l \leq k$, there exists a uniquely - k - colorable graph G with $gd_\chi(G) = l$.*

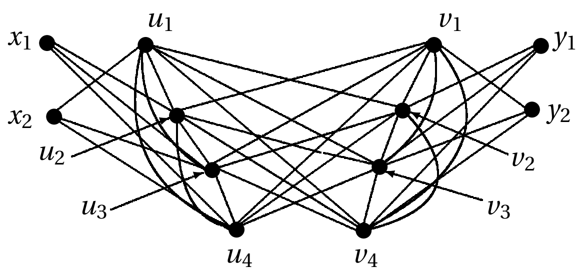

Figure 2 : A uniquely colorable graph with $gd_\chi = 2$ and $\chi = 4$.

Proof. For $l = 0$, take $G = K_k$. Assume $l \geq 1$. Then the required graph G is obtained from the complete k - partite graph with parts $V_1, V_2, \ldots, V_k$ where $V_i = \{u_i, v_i\}$, for all $i \in \{1, 2, \ldots, k\}$. Introducing $2l$ new vertices $x_1, x_2, \ldots, x_l$, $y_1, y_2, \ldots, y_l$. For each $i \in \{1, 2, \ldots, l\}$, join the vertex x_i to each vertex of u_j, where $j \neq i$ and $1 \leq j \leq k$; and join the vertex y_i to each vertex of v_j, where $j \neq i$ and $1 \leq j \leq k$. Let G be the resultant graph. For $l = 2$ and $k = 4$, the graph G is given in Figure 2. From the construction of G, it is clear that G is a uniquely - k - colorable graph and $\delta(G) = l - 1$. One can easily verify that $\mathscr{C} = \{V_1 \cup \{x_1, y_1\}, V_2 \cup \{x_2, y_2\}, \ldots, V_l \cup \{x_l, y_l\}, V_{l+1}, \ldots, V_k\}$ is a χ-coloring of G in which $V_1 \cup \{x_1, y_1\}, V_2 \cup \{x_2, y_2\}, \ldots, V_l \cup \{x_l, y_l\}$ are global dominating sets of G. Therefore $gd_\chi(G) \geq l$. Since $\delta(G) = l - 1$ and by Theorem 1.3, we have $gd_\chi(G) \leq l$. Thus $gd_\chi(G) = l$. □

Theorem 3.2. *For given integers a, b and c with $0 \leq a \leq b \leq c$, there exists a graph G for which $gd_\chi(G) = a$, $d_\chi(G) = b$ and $\chi(G) = c$ except when $a = 0$ and $b = 1$.*

Proof. If a, b and c are integers with $gd_\chi(G) = a$, $d_\chi(G) = b$ and $\chi(G) = c$, then by Lemma 2.4, we have $b \geq 2$ when $a = 0$. Conversely, suppose a, b and c are integers with $0 \leq a \leq b \leq c$ and $b \geq 2$ when $a = 0$. We construct the required graph G as follows.

Case 1. $a = 0$.

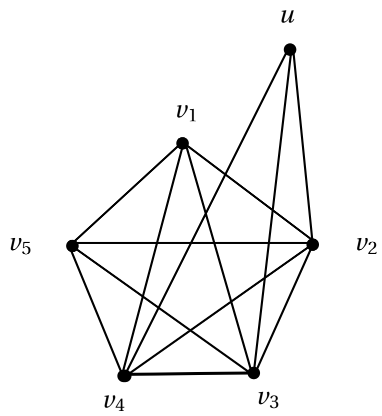

Figure 3 : A graph with $gd_\chi = 0$, $d_\chi = 4$ and $\chi = 5$.

Then by assumption $b \geq 2$. Consider the complete graph K_c on c vertices with the vertex set $\{v_1, v_2, \ldots, v_c\}$. Introduce a vertex u and join it to each of the vertices $v_2, v_3, ..., v_b$ by an edge. For $a = 0$, $b = 4$ and $c = 5$, the graph G is illustrated in Figure 3. Clearly $\chi(G) = c$. Since $\Delta(G) = n-1$, it follows from Theorem 1.5 that $gd_\chi(G) = 0$. Further, $\{\{v_1, u\}, \{v_2\}, \{v_3\}, \ldots, \{v_c\}\}$ is a χ-coloring of G where $\{v_1, u\}, \{v_2\}, \{v_3\}, \ldots, \{v_b\}$ are dominating sets of G so that $d_\chi(G) \geq b$. The inequality $d_\chi(G) \leq b$ follows from Theorem 1.2 as $\delta(G) = b-1$. Thus $d_\chi(G) = b$.

Case 2. $a \geq 1$.

Here, consider a complete c - partite graph $H = K_{\underbrace{2,2,\ldots,2}_{c\ times}}$ with parts $V_1, V_2, \ldots, V_c$ where $V_i = \{x_i, y_i\}$ for all $i \in \{1,2,\ldots,c\}$. Introduce $2a$ new vertices ; let them be $u_1, u_2, \ldots, u_a, v_1, v_2, \ldots, v_a$. For each $i \in \{1,2,\ldots,a\}$, join the vertex u_i to each vertex of the set $\{x_j : j \neq i \text{ and } 1 \leq j \leq b\}$. Similarly, for each $i \in \{1,2,\ldots,a\}$, join the vertex v_i to each vertex of the set $\{y_j : j \neq i \text{ and } 1 \leq j \leq b\}$. Let G be the resultant graph. For $a = 2$, $b = 4$ and $c = 5$, the graph G is illustrated in Figure 4. Clearly, $\chi(G) = c$. Now, consider the χ-coloring $\mathscr{C} = \{V_1 \cup \{u_1, v_1\}, V_2 \cup \{u_2, v_2\}, \ldots, V_a \cup \{u_a, v_a\}, V_{a+1}, V_{a+2}, \ldots, V_c\}$ of G. It is easy to verify that for each $i \in \{1,2,\ldots,a\}$, the set $V_i \cup \{u_i, v_i\}$ is a global dominating set of G and for each $j \in \{a+1, a+2, \ldots, b\}$, the set V_j is a dominating set of G. Hence $d_\chi(G) \geq b$ and $gd_\chi(G) \geq a$. By Theorem 1.2, we have $d_\chi(G) \leq b$ as $\delta(G) = b-1$ and thus $d_\chi(G) = b$. We now need to verify that $gd_\chi(G) \leq a$. Now, clearly the set $\{u_1, x_1, y_1, v_1\}$ is a global dominating set of G with minimum cardinality so that $\gamma_g(G) = 4$. Also $s(G) = 2$. Therefore by Theorem 1.4, we have $gd_\chi(G) \leq \frac{2a+2c-2c}{2} = a$. Hence $gd_\chi(G) = a$. □

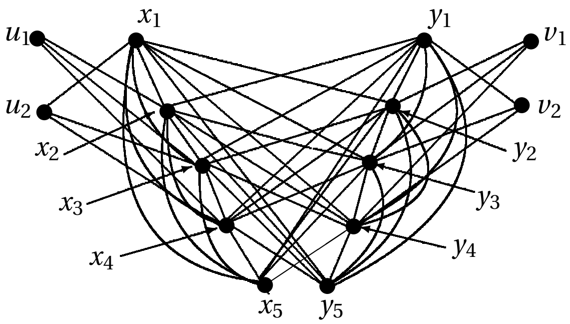

Figure 4 : A graph with $gd_\chi = 2$, $d_\chi = 4$ and $\chi = 5$.

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